Question 1 **(8 marks)**

Solve the following equations for x

(a)

Solution
$$\frac{3x}{2}=\frac{7}{5}$$ $$14=15x$$ $$x=\frac{14}{15}$$
Specific behaviours
✓ Cross multiply ✓ $\frac{14}{15}$

(b)

Solution
$$\frac{2x+6}{5}-\frac{x-3}{4}=2$$ $$(\frac{8x+24-(5x-15)}{20}=2$$ $$3x+39=40$$ $$x=\frac{1}{3}$$
Specific behaviours
✓ Common denominator ✓ Cross multiply ✓ $\frac{1}{3}$

(c)

Solution
$$x^3-2x^2-3x=0$$ $$x(x^2-2x-3)=0$$ $$x(x-3)(x+1)=0$$ $$x=0,3,-1$$
Specific behaviours
✓ Factorise ✓✓ $0,3,-1$

Question 2 **(5 marks)**

Solve the following equations for x

(a)

Solution
$-1 = \frac{3+x}{2}$ $-2 = \frac{-5+y}{2}$ $x = -5$ $y = 1$ C = (-5,1)
Specific behaviours
✓✓ $(-5,1)$

(b)

Solution
$m = \frac{-5-(-2)}{3-(-1)}$ $m = -\frac{3}{4}$ Perpendicular = $\frac{4}{3}$ $-5 = \frac{4}{3}(3) + c$ $c = -9$ $y = \frac{4}{3x} - 9$
Specific behaviours
✓ Correctly calculates gradient ✓ Correctly calculates perpendicular ✓ $y = \frac{4}{3x} - 9$

Question 3 **(11 marks)**

Consider the polynomial $P(x) = 2x^3 - 12x^2 + 22x - 12$

(a) State the degree of $P(x)$ (1 marks)

Solution
3rd Degree polynomial
Specific behaviours
✓ States the degree of the polynomial

(b) Show that $P(x)$ has an x intercept at (3,0) (2 marks)

Solution
$2(3)^3 - 12(3)^2 + 22(3) - 12 = 0$ $2(27) - 12(9) + 22(3) - 12 = 0$ $54 - 108 + 66 - 12 = 0$ $120 - 120 = 0$ $0 = 0\ (confirmed)$
Specific behaviours
✓ Substitutes into equation   ✓ Calculates correctly

(c) Show that $x - 1$ is a factor of $P(x)$ (2 marks)

Solution
$P(1) = 2(1)^3 - 12(1)^2 + 22(1) - 12$ $P(1) = 0$
Specific behaviours
✓ Substitutes into 1 into function – factor theorem   ✓ Calculates zero

(d) State $P(x)$ in **FULLY** factorized form (3 marks)

Solution
$P(x) = 2(x^3 - 6x^2 + 11x - 6)$ $P(x) = 2(x-1)(x-3)(x+a)$ $P(x) = 2(x^3 + (a-4)x^2 + (-4a+3)x + 3a)$ $-6 = 3a$ $a = -2$ $P(x) = 2(x-1)(x-3)(x-2)$
Specific behaviours
✓ Includes factors from part b and c i.e. $(x-1)(x-3)$ ✓ Factorise constant (2) out of function   ✓ States answer in Fully factored form

Question 4 **(8 marks)**

Given that $P(A|B') = \frac{4}{5}$, $P(B) = \frac{1}{8}$ and $P(A) = \frac{4}{5}$,

a) find $P(A \cap B)$. (3 marks)

Solution
$P(A \cap B') = P(A\|B')\,P(B)$ $= \frac{4}{5} \times \frac{7}{8}$ $= \frac{7}{10}$ $P(A \cap B) = P(A) - P(A \cap B')$ $= \frac{4}{5} - \frac{7}{10}$ $= \frac{1}{10}$
Specific behaviours
✓ determining $P(A \cap B')$ ✓ setting up expression for determining $P(A \cap B)$ ✓ determining value for $P(A \cap B)$

b) find $P(B|A')$. (3 marks)

Solution
$P(A') = \frac{1}{5}$ $P(B\|A') = \frac{P(A' \cap B)}{P(A')}$ $= \frac{P(B) - P(A \cap B)}{P(A')}$ $= \frac{\frac{1}{8} - \frac{1}{10}}{\frac{1}{5}}$ $= \frac{1}{8}$
Specific behaviours
✓ determining $P(A \cap B')$ ✓ setting up expression for determining $P(A \cap B)$

c) State, with a reason, whether A and B are independent events. (2 marks)

Solution
Yes. This is because $P(A \cap B) = P(A) \times P(B) = \frac{1}{10}$.
Specific behaviours
✓ response ✓ answer

Question 5 **(7 marks)**

a) Find the point of intersection of: 4 marks

$$y = x^2 - 4x + 2 \quad \text{and} \quad y = -x^2 - 8x$$

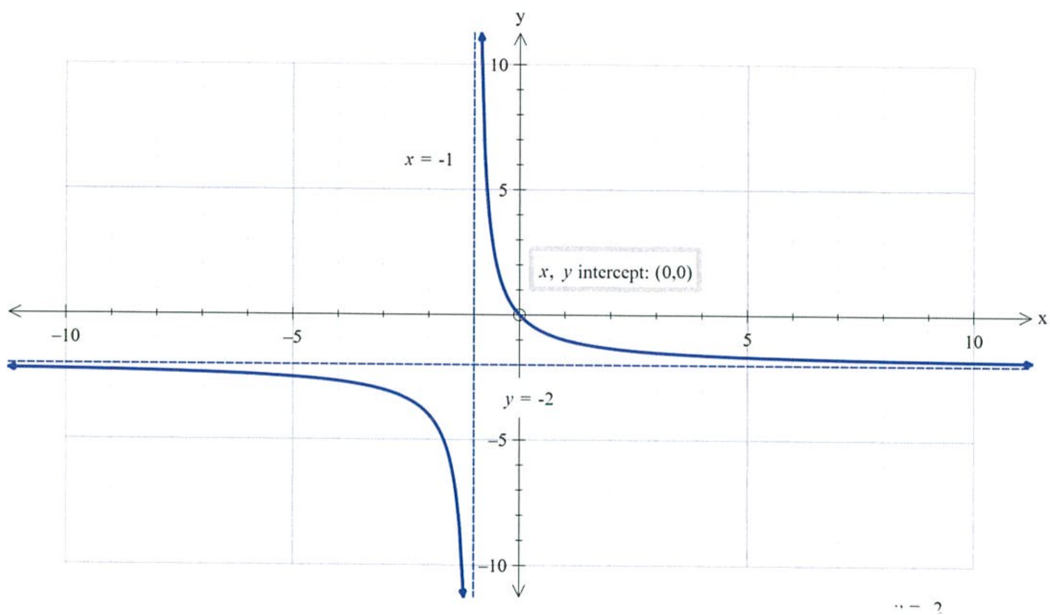
Solution
$x^2 - 4x + 2 = -x^2 - 8x$ $2x^2 + 4x + 2 = 0$ $2(x+1)^2 = 0 \rightarrow x = -1 \rightarrow y = (-1)^2 - 8(-1) = 7$ $\therefore$ intersection pt is at (-1,7)
Specific behaviours
✓ equating both quadratic equations ✓ factorising quadratic ✓ finding value for x ✓finding value for y

b) Solve $2(3x^2 - 5) - (x+2)(x-3) = 0$. 3 marks

Solution
$2(3x^2 - 5) - (x+2)(x+3) = 0$ $6x^2 - 10 - (x^2 - x - 6) = 0$ $5x^2 + x - 4 = 0$ $(5x-4)(x+1) = 0 \rightarrow x = -1, \frac{4}{5}$
Specific behaviours
✓ expanding and distributing ✓ expressing quadratic in factorised form ✓ finding the 2 solutions for x

Question 6 **12 marks**

a) Sketch graph of $y = \dfrac{2}{x+1} - 2$, labelling all special features. 5 marks

Solution

Specific behaviours
✓ shape ✓ accuracy ✓ ✓ vertical and horizontal asymptote ✓ x and y intercept at (0,0)

b)

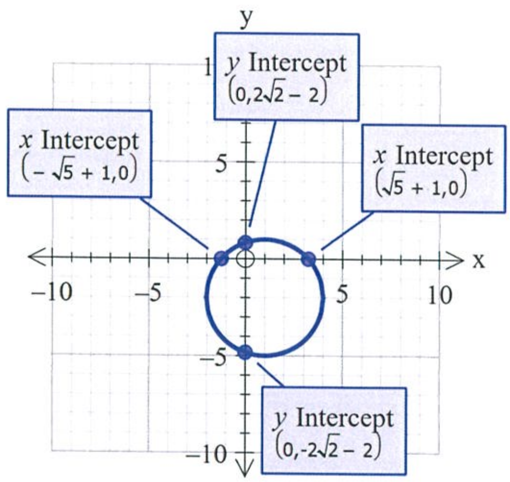
7 marks

i. Express $x^2 - 2x + y^2 + 4y - 4 = 0$ in the form $(x-h)^2 + (y-k)^2 = r^2$

Solution
$x^2 - 2x + y^2 + 4y - 4 = 0$ $x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4$ $(x-1)^2 + (y+2)^2 = 3^2$
Specific behaviours
✓ completing the square for x ✓ completing the square for y ✓ expressing in factorised form

ii. Hence sketch the graph of the circle. Label all intercepts with the axes.

Solution



Specific behaviours

✓ circle with centre at (1,-2) and radius of 3

✓ setting x=0 and y=0 to solve for intercepts

✓ x intercept , ✓ y intercept

Question 7 **6 marks**

AB is an interval. The coordinates of A and B are (2, 6) and (8, 6) respectively. Find:

a) the distance *AB* 2 marks

b) the midpoint of *AB* 2 marks

c) the equation of the circle with diameter *AB* 2 marks

Solution
a) distance $\overrightarrow{AB} = \sqrt{(8-2)^2 + (6-6)^2}$ $= \sqrt{36}$ $= 6$ units b) midpoint $= \left(\frac{2+8}{2}, \frac{6+6}{2}\right)$ $= (5,6)$ c) Equation of the circle $(x-5)^2 + (y-6)^2 = 9$
Specific behaviours
✓✓ setting up distance and finding the value ✓✓ setting up midpoint and finding the midpoint ✓✓ correct centre and radius

Question 8 **(11 marks)**

The curve C has equation $y = 4x^2 + 24x + A$, where A is a nonzero constant.

a) Express y in the form $p(x + q)^2 + r$. Hence, find the values for p and q, and an expression for r. 4 marks

Solution
$y = 4\left(x^2 + 6x + \frac{A}{4}\right)$ $= 4\left(x^2 + 6x + 9 + \frac{A}{4} - 9\right)$ $= 4(x + 3)^2 + A - 36$ $\therefore\ p = 4,\ q = 3,\ r = A - 36$
Specific behaviours
✓ expressing quadratic in turning point form ✓ finding p ✓ finding q ✓ expression for r.

b) A straight line L has an equation $y = Bx + 10$, where B is a nonzero constant. Given that C and L meet at the points with $x = -1$ and $x = -\frac{21}{4}$, determine the values of A and B. 7 marks

Solution

for the point of intersection:

$$4x^2 + 24x + A = Bx + 10$$
$$4x^2 - x(B - 24) + A - 10 = 0$$

when $x = -1$

$$4(-1)^2 - (-1)(B + 24) + A - 10 = 0$$
$$A + B = 30 \rightarrow 1$$

when $x = -\frac{21}{4}$

$$4\left(-\frac{21}{4}\right)^2 - \left(-\frac{21}{4}\right)(B - 24) + A - 10 = 0$$
$$4A + 21B = 103 \rightarrow 2$$

Solving simultaneously for the values of A and B from equations 1 and 2

$$4(30 - B) + 21B = 103$$
$$B = -1 \rightarrow A = 31$$

Specific behaviours

✓ equating C and L

✓ substituting -1

✓ finding eq 1

✓ substituting $-\frac{21}{4}$

✓finding eq 2

✓value for B

✓ value for A

Question 9 **(10 marks)**

Given $P(x) = -5x^2 - 6$ and $Q(x) = x + 1$ and $R(x) = 5x^2 + 3x$

(a) Simplify $P(x) + Q(x) + R(x)$ (2 marks)

Solution
$4x - 5$
Specific behaviours
✓ Substitutes correctly ✓ Simplifies answer

(b) Simplify $Q(x) - P(x)$ (2 marks)

Solution
$5x^2 + x + 7$
Specific behaviours
✓Substitutes correctly ✓ Simplifies answer

(c) Simplify $P(x) \times R(x)$ (2 marks)

Solution
$-25x^4 - 15x^3 - 30x^2 - 18x$
Specific behaviours
✓ Multiplies binomials with distributive law ✓ Simplifies answer

(d) Simplify $P(x) - Q(x) - R(x)$ (2 marks)

Solution
$-10x^2 - 4x - 7$
Specific behaviours
✓ Distributes negatives correctly ✓ Simplifies answer

(e) Simplify $R(x) - Q(x)P(x)$ (2 marks)

Solution
$5x^2 + 10x^2 + 9x + 6$
Specific behaviours
✓ Correct order of operations ✓ Simplifies answer

(e) State all solutions to $P(x) = 0$ (3 marks)

Solution
$x = 1$ $x = 2$ $x = 3$
Specific behaviours
✓ per each solution

Question 10 **(8 marks)**

A box contains 35 apples, of which 25 are red and 10 are green. Of the red apples, five contain an insect and of the green apples, one contains an insect. Two apples are chosen at random from the box. Find the probability that:

a) both apples are red and at least one contains an insect. (3 marks)

Solution
$P(red \text{ apples,at least one with an } insect) = P(RWI,RNI) + P(RWI,RWI) + P(RNI,RWI)$ $= \frac{5}{35} \times \frac{20}{34} + \frac{5}{35} \times \frac{4}{34} + \frac{20}{35} \times \frac{5}{34}$ $= \frac{22}{119}$
Specific behaviours
✓ ✓ setting up expression for probability   ✓ answer

b) at least one apple contains an insect given that both apples are red. (2 marks)

Solution
$P(WI,R\ R) = \frac{P(WI \cap R\ R)}{P\ (R\ R)}$ $= \frac{\frac{22}{119}}{\frac{25}{35} \times \frac{24}{34}}$ $= \frac{11}{30}$
Specific behaviours
✓ setting up expression for conditional probability   ✓ answer

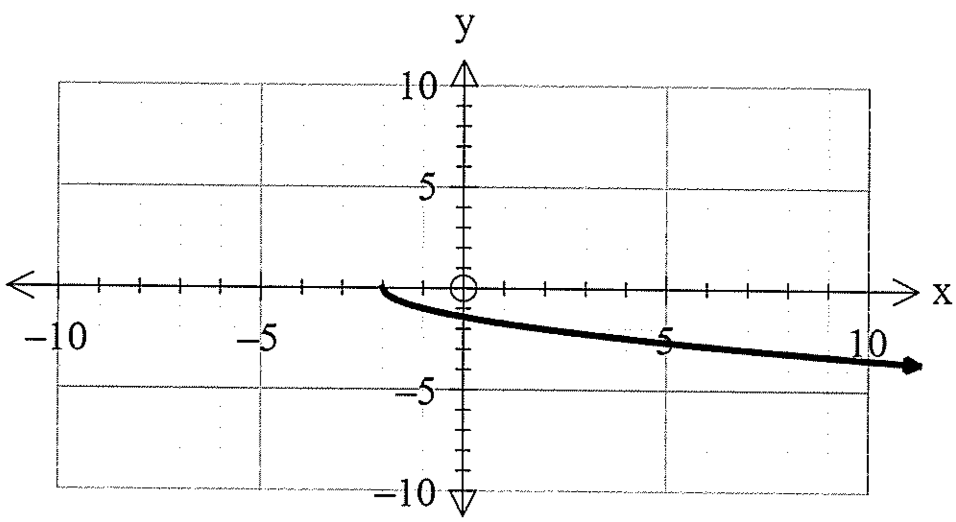
c) both apples are red given that at least one is red. (3 marks)

Solution
$P(R\ R \mid \text{at least one } red) = \dfrac{\dfrac{25}{35} \times \dfrac{24}{34}}{1 - \left(\dfrac{10}{35} \times \dfrac{9}{34}\right)}$ $= \dfrac{6}{11}$
Specific behaviours
✓ setting up numerator   ✓ setting up denominator   ✓ answer

QUESTION 11 [10 marks]

Determine the equations of the following graphs:

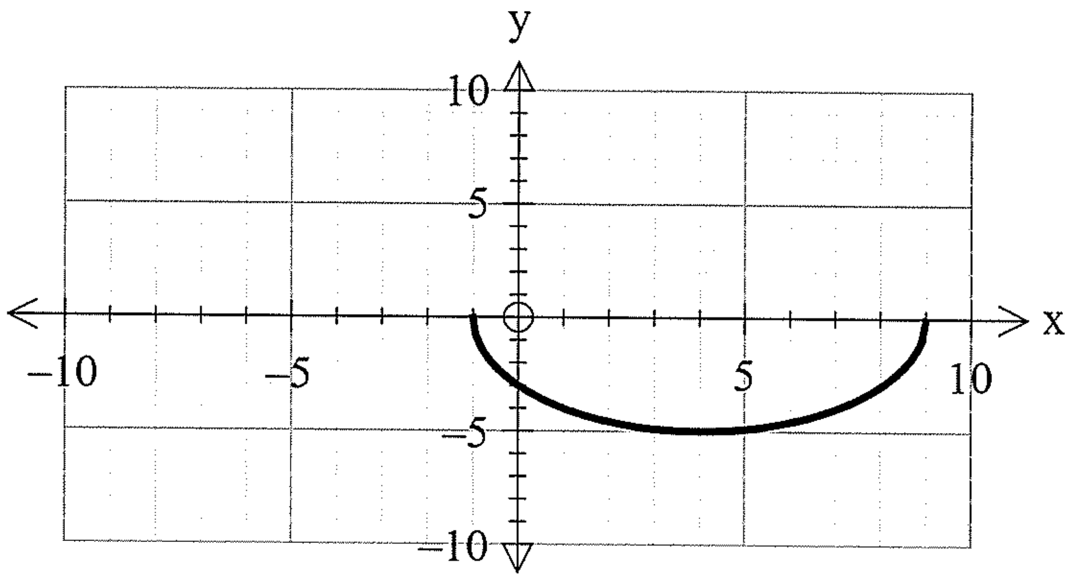
a) 2 marks



Solution
$y = -\sqrt{x+2}$
Specific behaviours
✓ -, ✓ +2

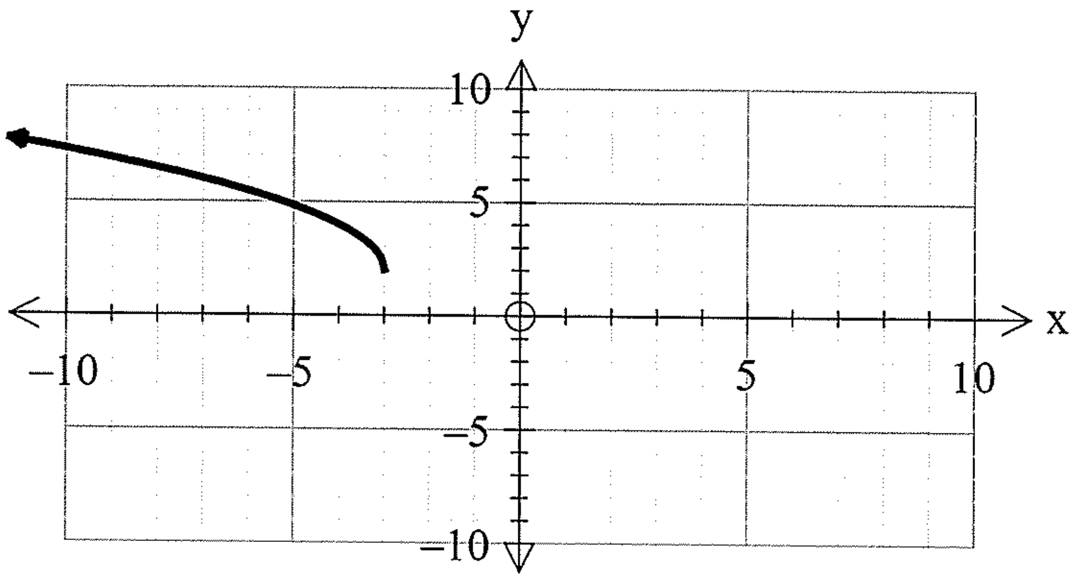
b)

4 marks



Solution
$y = -\sqrt{25-(x-4)^2} - 3$
Specific behaviours
✓ - ,✓ 25 , ✓ -(x-4), ✓-3

c) 4 marks



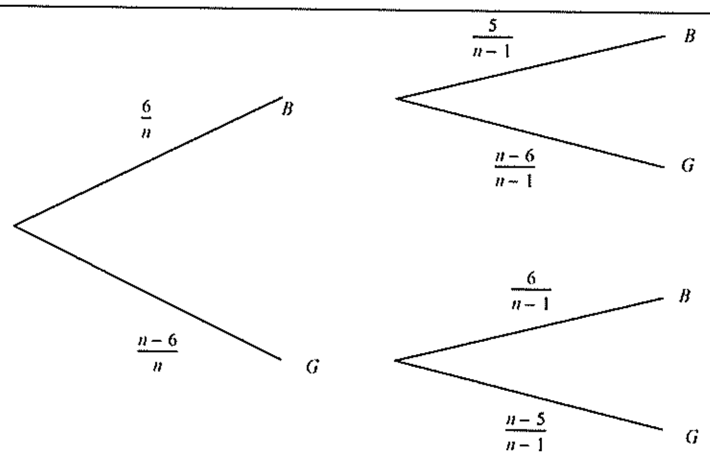
Solution
$y = 2\sqrt{-(x+3)} + 2$
Specific behaviours
✓ 2 ,✓ - , ✓ -(x+3), ✓+2

Question 12 **(10 marks)**

There are n beads in a bag. Six of them are green and the rest are blue. Jon picks one bead out of the bag and does not replace it. He then picks another bead at random.

a) Represent the situation above by drawing a tree diagram. (2 marks)

Solution



Specific behaviours

✓ branches set up

✓ correct labels for the branches

b) The probability of picking 2 blue beads is $\frac{1}{3}$. Show that $n^2 - n - 90 = 0$. (3 marks)

Solution

$$P(\text{ blue, blue}) = \frac{6}{n} \times \frac{5}{n-1}$$

$$\frac{6}{n} \times \frac{5}{n-1} = \frac{1}{3}$$

$$\frac{30}{n^2 - n} = \frac{1}{3}$$

$$90 = n^2 - n$$

$$n^2 - n - 90 = 0$$

Specific behaviours

✓ setting up probability for 2 blue beads and equating to 1/3

✓ cross multiplying

✓ equating correct quadratic equation to zero

c) How many beads are in the bag? (2 marks)

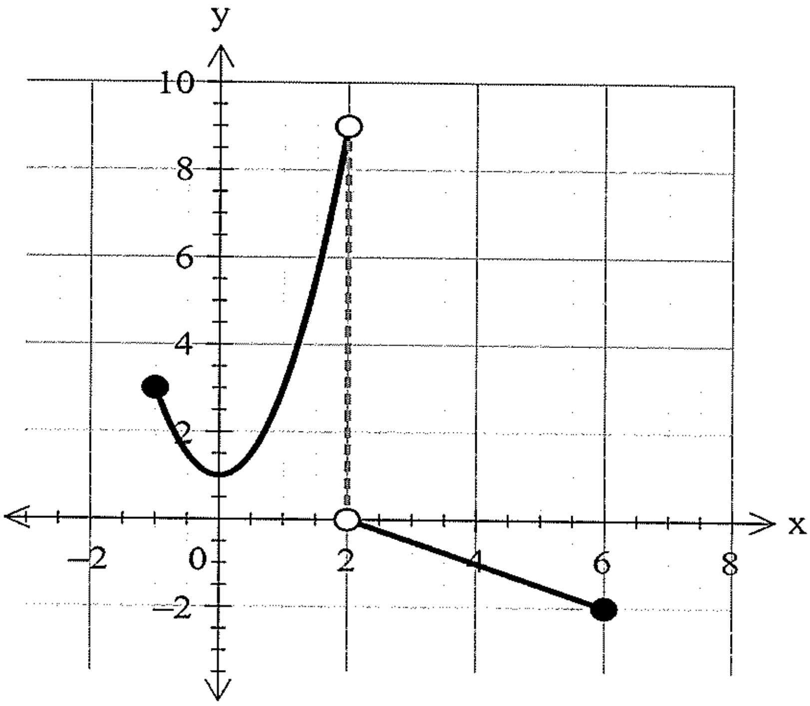
Solution
$(n-10)(n+9)=0$ $n = 10$ or $n = -9$ disregard $n = -9$ ∴ there are 10 beads in the bag
Specific behaviours
✓ factorising quadratic ✓ stating that there are 10 beads in the bag

d) Find the probability of picking 2 beads of different colours. (3 marks)

Solution
P(different colours)= $P(B,G) + P(G,B)$ $= \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{6}{9}$ $= \frac{8}{15}$
Specific behaviours
✓ ✓ setting up probability for each of the different colours ✓ simplified fraction

Question 13 **14 marks**

a) Determine the equation of the following piece-wise defined function below. (4 marks)



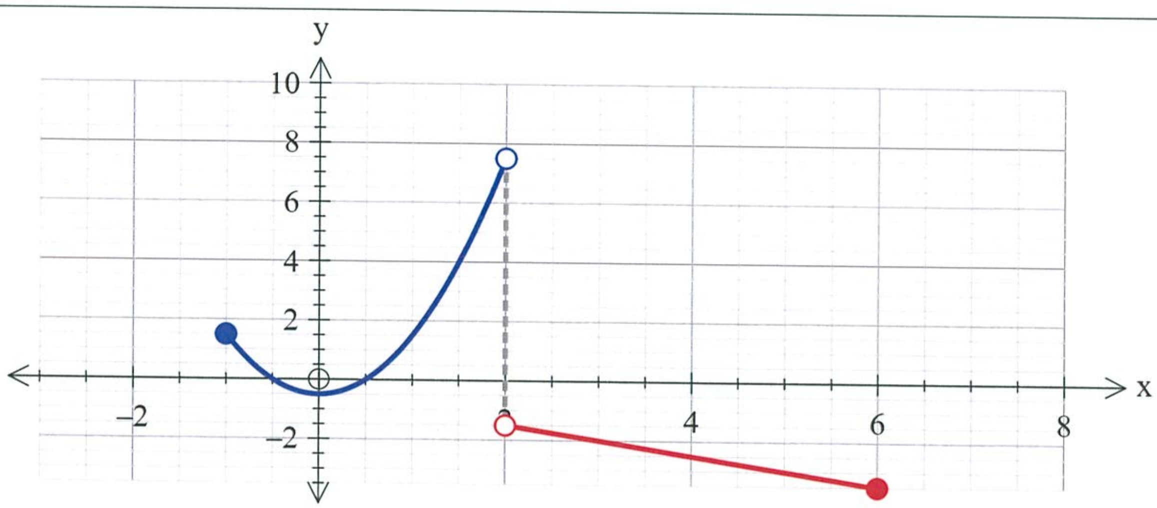
Solution
$f(x) = \begin{cases} 2x^2 + 1 & -1 \leq x < 2 \\ -\frac{1}{2}x + 1 & 2 < x \leq 6 \end{cases}$
Specific Behaviours
✓ ✓writing equations ✓ ✓ writing domain

b) State the domain and range of the function. (2 marks)

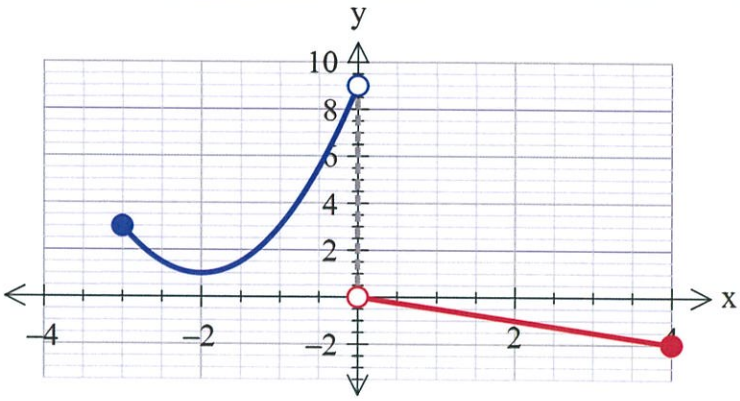
Solution
$D_x = \{x : -1 \leq x \leq 6 \text{ or } x \neq 2 \quad x \in \mathbb{R}\}$ $R_x = \{ y : -2 \leq y < 0 \text{ or } 1 \leq y < 9 \quad y \in \mathbb{R}\}$
Specific behaviours
✓ domain   ✓ range

c) On the axes provided sketch the following. (8 marks)

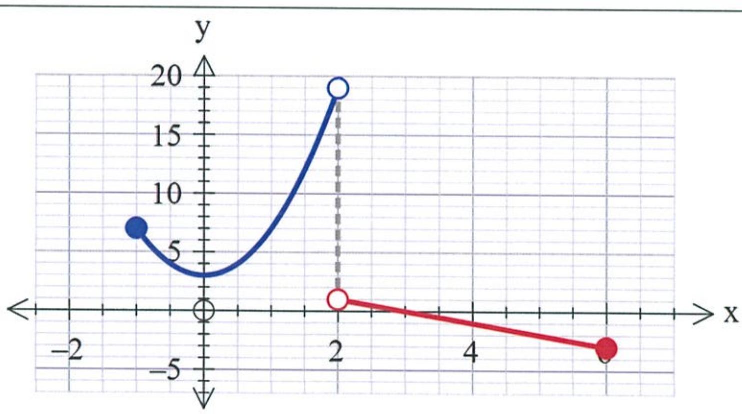
i) $f(x) - 1{\cdot}5$

Solution

Specific behaviours
✓ shape   ✓ accuracy

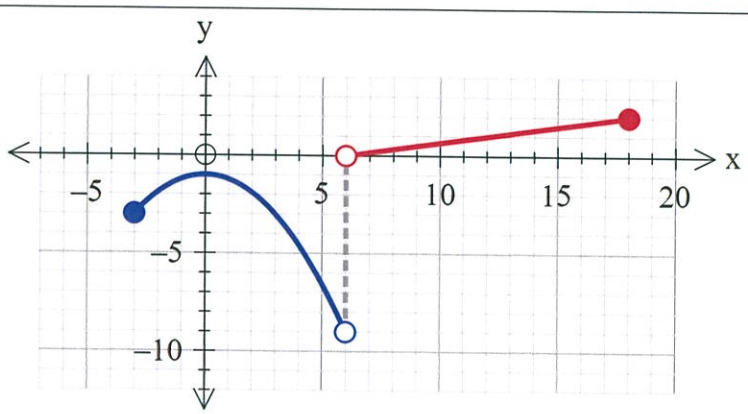
ii) $f(x + 2)$

Solution

Specific behaviours
✓ shape ✓ accuracy

iii) $2f(x) + 1$

Solution

Specific behaviours
✓ shape ✓ accuracy

iv) $-f\left(\frac{1}{3}x\right)$

Solution

Specific behaviours
✓ shape ✓ accuracy

Question 14 **5 marks**

A quadratic function has the equation $f(x) = 2x^2 + 4x - 6$. (3 marks)

a) Find the value of p for which the equation $f(x) + p = 0$ has one solution.

Solution
$f(x) = 2(x^2 + 2x - 6)$ $= 2(x + 1)^2 - 8$ turning point: (-1,-8) for one solution, $p = 8$
Specific behaviours
✓ writing quadratic equation in turning point form ✓ stating turning point ✓ finding value for p

b) Find the value of q for which $f(x - q) = 0$ (2 marks)

i) two positive solutions.

Solution
$q > 3$
Specific behaviours

ii) two negative solutions.

Solution
$q < -1$
Specific behaviours

Question 15 **8 marks**

Find the natural Domain and Range of these functions:

a) $f(x) = -3x^2 + 6x - 8$

b) $g(x) = \sqrt{3x + 5}$

c) $h(x) = -\sqrt{5^2 - (x-2)^2}$

d) $k(x) = \dfrac{2}{x^2 - 1}$

Solution
a. D_x is $\mathbb{R}$, $R_x = (-\infty, -5)$ b. $D_x = \left[\frac{5}{3}, \infty\right)$ $R_x = [0,\infty)$ c. $D_x = [-3,7]$ $R_x = [0,-5]$ d. $D_x = (-\infty,-1) \cup (-1,1) \cup (1,\infty)$ $R_x = (-\infty,-2] \cup (0,\infty)$
Specific behaviours
✓ each for the domain ✓ each for the range

Question 16 **(12 marks)**

Modern Corporation produces three products where the cost function C in terms of the number of items produced q and for $0 \leq q \leq 50$ is given by:

Product 1 $C(q) = \frac{q^2}{10} + 5q + 16$

Product 2 $C(q) = 500 + 43q - 7q^2 + q^3$

Product 3 $C(q) = q + \sqrt{q + 1} + 200$

(a) Determine (6 marks)

i) the fixed costs involved in the production of each product.

ii)the total cost of producing 50 units of each product.

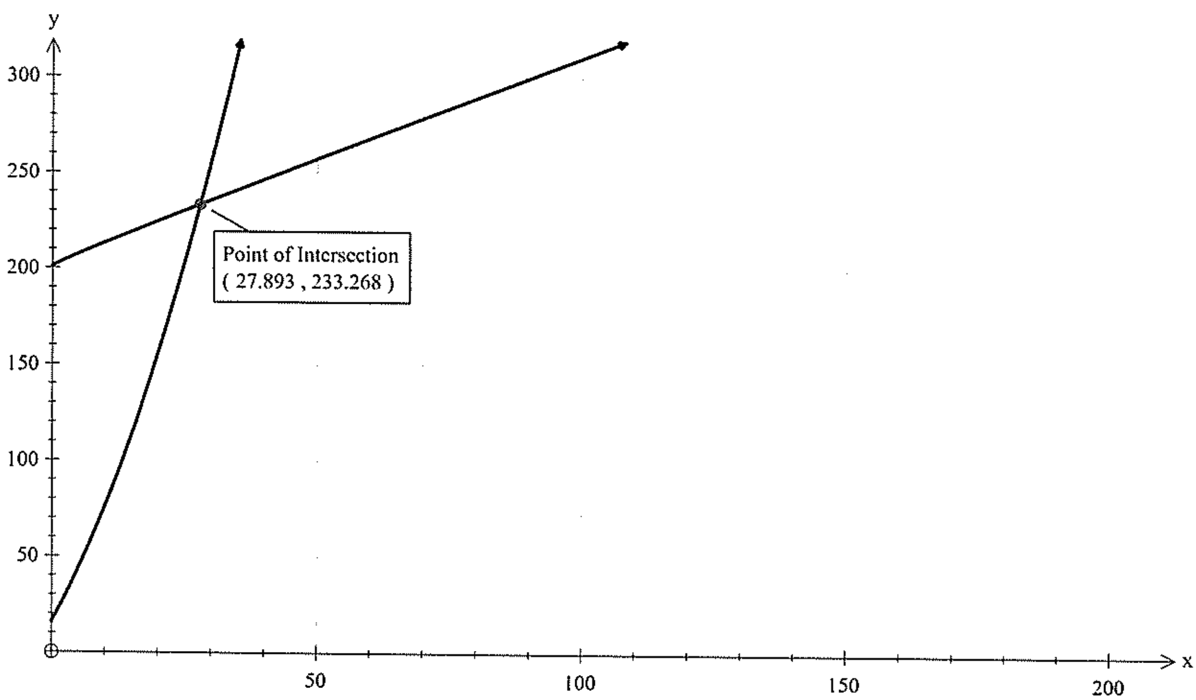
Solution
$C(0) =$ \$16 per product 1 $C(0) =$ \$ 500 per product 2 $C(0) =$ \$200 per product 3 $C(50) = \frac{50^2}{10} + 5 \times 50 + 16 =$ \$516 $C(50) = 500 + 43 \times 50 - 7 \times 50^2 + 50^3 =$ \$110 150 $C(50) = 50 + \sqrt{50 + 1} + 200 =$ \$257.14
Specific behaviours
✓ each for fixed cost, C(0) ✓ each for cost of producing 50 units, C(50)

(b) 3 marks.

Approximately how many of each product, need to be produced so that the cost of production is $240?

Solution
$\frac{q^2}{10} + 5q + 16 = 240$ $q = 28.526$, $\approx$ 29 units $500 + 43q - 7q^2 + q^3 = 240$ $q = -3.35$, no solution $q + \sqrt{q+1} + 200 = 240$ $q = 34.077$, $\approx$ 34 units
Specific behaviours
✓ each for units

(c) Will the cost of production of products 1 and 3 ever be the same for a specified value of q. If so, determine the approximate value of q and the total cost of production where this occurs.

Solution
 28 units at $233.27
Specific behaviours

Question 17 **5 marks**

Show that the circles $x^2 + y^2 - 2x - 3y = 0$ and $x^2 + y^2 + x - y = 6$ intersect on the x-axis and y-axis.

Solution
When $x = 0$, $y^2 - 3y = 0 \rightarrow$ y intercepts at (0,0) and (0,3) $y^2 - y - 6 = 0$ $(y-3)(y+2) = 0 \rightarrow$ y intercepts at (0,3) and (0,-2) When $y = 0$ $x^2 - 2x = 0 \rightarrow$ x intercepts at (2,0) and (0,0) $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0 \rightarrow$ x intercepts at (-3,0) and (2,0) The y intercepts are common at (0,3) and x intercepts common at (2,0). Therefore both circles intersect at the x axis and the y axis.
Specific behaviours
✓ Setting x =0 for both equations ✓finding the y intercepts for both circles ✓setting y=0 for both equations ✓finding the x intercepts for both circles ✓justification

Question 18 **(3 marks)**

(a)

Travelling at an average speed of 60km/h Dr George takes 15 minutes to reach his surgery. If he wishes to reach his surgery three minutes faster, by how much must he increase his average speed?

Solution
$s = \frac{d}{t}$ $d = 15 \times \frac{15}{60}$ $\therefore s = \frac{15 \times \frac{15}{4}}{\left(\frac{12}{60}\right)}$ $= 75$ km/h ∴ Dr George has to increase his speed by 15 km/h.
Specific behaviours
✓ Calculate distance correctly ✓ 75 km/h ✓ 15 km/h